Mixed Oligopoly in a Two-dimensional Space

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Abstract
This paper extends a mixed oligopoly model developed by Matsushima and Matsumura (MM) into a two-dimensional space with one public firm and two private firms. By doing so, another type of equilibrium is established: In one dimension the private firms choose the same location, whereas in the other dimension they are located equidistant from the center in the opposite direction from each other. We found that when the public firm moves toward the private firms with lower transport costs, the private firms approach the public firm in one dimension. On the other hand, the private firms move away from the public firm in the other dimension. This is in sharp contrast to full agglomeration in MM, where private firms just move away from an approaching public firm.

1. Introduction

Hotelling's (1929) seminal work showed that duopolistic firms agglomerate in the center of a one-dimensional space (a linear city) although d'Aspremont et al. (1979) revised the result as maximum differentiation under Bertrand spatial competition. As another branch of research, Hamilton et al. (1989) and Anderson and Neven (1991) developed location-then-quantity games, which showed the agglomeration of firms in the center (minimum differentiation).

Since the appearance of a study by Merrill and Schneider (1966), mixed oligopoly that has different objectives of agents in a market (typically, there are private and public sectors) has been a big issue in theory and practice.[1] The synthesis of location and mixed oligopoly theories has been developed as a result. For example, Cremer et al. (1991) analyzed location-price competition with the public sector.

Matsushima and Matsumura (2003, henceforth, MM) focused on an observation that in mixed markets private firms often provide a different good or service from those offered by the public sector, although the offering might be similar to that of other private firms' (herd behavior of private firms).[2] To explain their observation, MM incorporated a welfare-maximizing public firm into a spatial Cournot model. As a result, in their circular city all private firms agglomerate at a point that is the farthest from the public firm.[3] In their linear city represented by the line segment, there are two types of equilibrium: (i) All private firms agglomerate near the edge (at around 1/10), whereas the public firm locates near the center (1/2), or (ii) Two locations that are symmetric with regard to the center (specifically, 1/10 and 9/10) have half of the private firms each, and the public firm locates at the center if and only if the number of the private firms is even.

In sum, a public firm has a strong repelling effect against private firms because the public firm sets the price in each market at its transport cost in that location (marginal cost pricing) to maximize the social surplus. Hence, markets near the public firm are very competitive with the private firms. Because each price is effectively determined by the location of the public firm, optimal location choices for private firms that have the

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same cost conditions become identical. Consequently, they often agglomerate.

In this study we extended MM into a two-dimensional space. In studies with two-dimensional Cournot competition but without a public sector, Berenguer-Maldonado et al. (2005) showed that firms agglomerate in the center when space is a disk.[4] Because this agglomeration result is essentially identical as in one-dimensional space (Hamilton et al., 1989 ; Anderson and Neven, 1991), one may think that dimensionality hardly matters. However, at least in Bertrand spatial competition, dimensionality has an important role. In their location-then-price competition in a multi-dimensional space, Tabuchi (1994) and Irmen and Thisse (1998) show that maximum differentiation occurs in only one dimension, while minimum differentiation is achieved in each of the other dimensions. Hence, we cannot directly predict from a one-dimensional case how firms will react against their rivals in location. For this reason, we tackled the case of a mixed oligopoly model in two-dimensional space.

Another reason for the expansion of dimensions is from a casual observation in the real world. MM suggested "Television programs supplied by the Japan Broadcasting Corporation (NHK) are quite different from those of private broadcasting companies, which are quite similar to each other." It seems true that a public broadcasting company would prefer a more serious program like news to an entertainment or a shopping program, which seems to be more suitable for private broadcasting companies. However, to be politic, a public company may choose, or be effectively required to choose, a moderate course, while positions of private companies often differ: e.g., they may feature conservative or liberal content. Such a complex positioning, which we will see as partial agglomeration below, can be analyzed by a multi-dimensional model.

The remainder of this paper is organized as follows. In Section 2, the two-stage location-then-quantity game is presented. In Section 3, quantity choice is analyzed in the second stage. In Section 4, we analyze location equilibria in the first stage. Section 5 summarizes the results of our analysis.

2. The model

Our model is based on a one-dimensional model developed by Matsushima and Matsumura (2003), who analyzed location-then-quantity competition with a welfare-maximizing public firm and profit-maximizing private firms. We extend the analysis to a two-dimensional model with simplification in two ways: First, the functional form of transport costs is assumed to be quadratic in Euclidean distance, and second, the number of private firms is two. Except for those aspects, the MM model and this model are identical so they can be compared easily.

We consider a two-dimensional city expressed by a square on the x-y coordinates, $L=\{(x, y) \in \mathbb{R}^2 : -1 \leq x \leq 1, -1 \leq y \leq 1\}$, and consumers are uniformly and continuously distributed on L with a density of one at each location. There are a welfare-maximizing public firm (firm 0) and two profit-maximizing firms (firm 1 and firm 2) that supply a homogeneous good with zero marginal cost. Let the location of firm i be $(x_i, y_i) \in L$.

In the first stage, each firm simultaneously chooses its location. In the second stage, each firm simultaneously chooses its quantity given firms' locations. We use subgame perfection as the equilibrium concept. Based on the literature, firms bear transport costs, and they can set a supply amount for each location independently because arbitrage between consumers is assumed to be prohibitively costly.

Each consumer has the same inverse demand function, as follows:

$$P(x, y) = a - bQ(x, y),\ Q(x, y) \equiv \sum_{i=0}^{2} q_i(x, y), \tag{1}$$

where $P(x, y)$ is the price at (x, y), $q_i(x, y)$ and $Q(x, y)$ are the supply amount of each firm and the total supply amount there, respectively, and a and b are positive constants.

The transport costs are the same for the firms and are linear to supply amounts. The unit transport cost is quadratic with regard to Euclidean distance between a firm and a consumer. Let $d_i(x, y)$ denote the distance between a consumer at (x, y) and firm i. Then, the transport cost function is given by

$$td_i(x, y)^2 = t[(x_i - x)^2 + (y_i - y)^2],$$

where t is the transport cost parameter and is assumed to be sufficiently low, such that

$$a > 24t, \tag{2}$$

which ensures that the public firm always serves the entire city, irrespective of any locations of the firms.

For private firm i, the local profit earned at (x, y) and the total profit are respectively given by

$$\pi_i(x, y) = q_i(x, y)\ [P(x, y) - td_i(x, y)^2],$$

$$\Pi_i = \iint_L \pi_i(x, y)\,dxdy,$$

where $P(x, y)$ is given by (1). Let $w(x, y)$ denote the local social surplus (consumer surplus plus the sum of the profits) at (x, y), and then we get

$$w(x,y)=\int_0^{Q(x,y)}(a-bm)dm-\sum_{i=0}^{2}td_i(x,y)^2\,q_i(x,y)],$$

and the (total) social surplus is

$$ss=\iint_L w(x,y)dxdy.$$

3. Quantity choice

By backward induction, we analyze the second-stage quantity game under given locations. In this stage, we can apply the same analysis in MM because only distance matters, irrespective of dimensionality. Therefore, some important results in MM are clearly valid in our model as well; we readily have the following results.

Result 1 (*Lemma 1 in MM*) *In equilibrium*,

$$P(x,y)=td_0(x,y)^2,\ Q(x,y)=\frac{a-td_0(x,y)^2}{b} \tag{3}$$

Result 2 (*Lemma 2 in MM*) *Consumer surplus, cs, does not depend on* (x_i, y_i) $(i=1, 2)$ *and is given by*

$$\begin{aligned}\mathrm{cs}&=\iint_L cs(x,y)dxdy\\&=\frac{2}{45b}[45a^2-30at(2+3x_0^2+3y_0^2)\\&\quad+t^2(28+45x_0^4+45y_0^4+120x_0^2+120y_0^2+90x_0^2y_0^2], \end{aligned}\tag{4}$$

where $cs(x,y)$ *is consumer surplus at* (x,y):

$$cs(x,y)=\frac{[a-td_0(x,y)^2]^2}{2b}.$$

Result 3 (*Lemmas 3 and 4 in MM*) *Firm i* $(i=1, 2)$ *supply positive amount of* $q_i(x,y)$ *at* (x,y) *if and only if* $d_0(x,y)>d_i(x,y)$. *Then*,

$$q_i(x,y)=\frac{t\,[d_0(x,y)^2-d_i(x,y)^2]}{b},$$

and its profit is given by

$$\pi_i(x,y)=b[q_i(x,y)]^2.$$

Result 4 (*Lemma 5 in MM*) *The profit of each private firm does not depend on the location of the other private firm. And the total profit of firm i* $(i=1, 2)$ *is given by*

$$\Pi_i(x_0,y_0,x_i,y_i)=\iint_{L_i}\pi_i(x,y)dxdy, \tag{5}$$

where L_i $(i=1, 2)$ *denotes the domain in which firm i serves:*

$$L_i=\{(x,y):d_0(x,y)>d_i(x,y)\}\cap L.$$

In Result 2, note that if we rewrite (x_0, y_0) as $(r\cos\theta, r\sin\theta)$, then we have

$$cs=2[45a^2-30at(2+3r^2)+t^2(28+120r^2+45r^4)]/45b.$$

Because $\partial cs/\partial\theta=0$ and $\partial cs/\partial r=60tr[-3a+(4+3r^2)t]<0$ (the inequality is due to (2)), the nearer to the center the public firm locates, the greater *cs* becomes. In Result 4, we find that the market boundary of $d_0(x,y)=d_i(x,y)$ is the perpendicular bisector of the line segment joining the locations of firm 0 and firm *i*.

4. Location equilibrium

We analyze the first-stage game given the results in the second stage. Without loss of generality, we henceforth assume that $0\leq x_0,\ y_0\leq 1$. Furthermore, because of symmetry, if $(x_0, y_0, x_1, y_1, x_2, y_2)=(0, 0, 1/2, 1/2, -1, -1)$ were an equilibrium, then a rotationally symmetric location pair, e. g., $(0, 0, -1/2, -1/2, -1, 1)$, must be another equilibrium. For notational convenience, we will omit such other symmetric equilibria.

First of all, symmetry yields the following result.

Lemma 1 *The public firm locates on the x-axis (the y-axis) if and only if the private firms locate equidistantly away from the x-axis (the y-axis) in opposite directions from each other. In other words, in equilibrium*, $x_0=0\Leftrightarrow x_1+x_2=0$, *and* $y_0=0\Leftrightarrow y_1+y_2=0$.

Proof. See Appendix A.

This lemma classifies the types of location equilibria. For the public firm to choose the center, the private firms must locate symmetrically with regard to the center. Let this case be *central symmetry*, which will be analyzed in Subsection 4. 1 below.

On the other hand, unless the public firm locates on either of two axes, symmetry breaks down. From Result 4, we see that the profit functions of both private firms are only dependent on their own location and the location of the public firm; hence, both firms would choose the same location, $(x_1, y_1)=(x_2, y_2)$. We name this case *full agglomeration*, which appears in Subsection 4. 2.

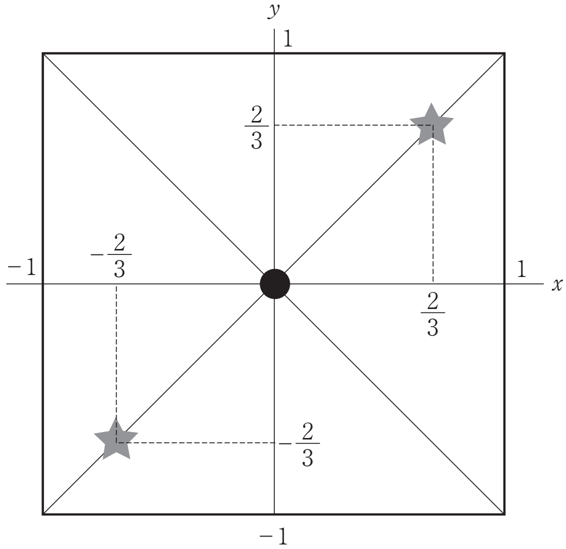
At last, there is another possibility: the public firm is located on only one axis. For example, suppose that $x_0=0$ and $y_0\neq 0$. In this case, $x_2=-x_1$ must hold, but the symmetry with regard to the *y*-coordinate has broken down. Again from Result 4, we see that the private firms would locate the same position with regard to the *y*-coordinate. We call this *partial agglomeration*. Subsection 4. 3 will demonstrate this case.

Central symmetry and full agglomeration were also presented in the linear-city case of MM. On the other hand, partial agglomeration is new and only derived by a multi-dimension model.

4. 1 Central symmetry

First, we focus on the possibility of central symmetry. Under this scheme, we have the location equilibrium as follows.

Figure 1. (Central symmetry): The circle represents the public firm; two stars show the private firms.



Proposition 1 *Under the scheme of central symmetry, there exists a unique location equilibrium such that*

$$(x_0, y_0, x_1, y_1, x_2, y_2) = (0, 0, -2/3, -2/3, 2/3, 2/3).$$

Proof. See Appendix B.

Figure 1 shows the equilibrium,[5] which corresponds to one of the equilibria in MM (Proposition 4 (ii) in MM): the public firm locates at 1/2, half of the private firms locate at 1/10 and the others locate at 9/10 in a linear city of a line segment [0, 1].

Unlike Tabuchi (1994) and Irmen and Thisse (1998), private firms differentiate with regard to all dimensions here. As Irmen and Thisse (1998) indicated, the key is the length of the market boundary between the public firm and the private firm. Roughly speaking, the length corresponds to the price elasticity of demand. The shorter the boundary, the less the loss of demand when a firm places higher prices on its good (low price elasticity). In our model, when $(x_0, y_0, x_1, y_1) = (0, 0, -2/3, -2/3)$, the length of the market boundary between firm 0 and firm 1 is $4\sqrt{2}/3 \approx 1.88562$. On the other hand, if $(x_0, y_0, x_1, y_1) = (0, 0, -1, 0)$, which represents differentiation in one dimension only, the length of the market boundary is 2. Therefore, the elasticity is lower in differentiation in both dimensions as in Proposition 1.

It is true that locating at $(x_1, y_1) = (-1, -1)$ minimizes the boundary when $(x_0, y_0) = (0, 0)$. However, Cournot competition has a relatively stronger centripetal force (Hamilton et al., 1989; Anderson and Neven, 1991). Consequently, such trade-off yields our interior location equilibrium.

4. 2 Full agglomeration

We analyze the case of full agglomeration. Remember that consumer surplus depends on a $(\partial cs/\partial a > 0)$, but the profits of the private firms do not (Result 2 and Result 4). Hence, when a is sufficiently large, effectively the public firm only cares about the consumer surplus. As a result, when $a \to \infty$, the public firm chooses $(x_0, y_0) = (0, 0)$. When $(x_0, y_0) = (0, 0)$, as Subsection 4. 1 shows, a private firm would choose $(x_1, y_1) = (-2/3, -2/3)$. Because of independence from the other firm's location with regard to affecting its profit, we have the following.

Proposition 2 *Under the scheme of full agglomeration, we have*

$$\lim_{a\to\infty} (x_0, y_0, x_1, y_1, x_2, y_2) = (0, 0, -2/3, -2/3, -2/3, -2/3).$$

Figure 2 illustrates the situation. However, from Lemma 1, we know that $(x_0, y_0) = (0, 0)$ is consistent only in the case of *central symmetry*. Hence, we conduct a comparative statics in the vicinity of

$$(x_0, y_0, x_1, y_1, x_2, y_2) = (0, 0, -2/3, -2/3, -2/3, -2/3).$$

Let

$$0 < \tau \equiv t/a < \frac{1}{24}, \tag{6}$$

then the solutions depend only on τ. From the first-order conditions, $\partial ss/\partial x_0 = 0$, $\partial ss/\partial y_0 = 0$, $\partial \Pi_i/\partial x_i = 0$ and $\partial \Pi_i/\partial y_i = 0$ $(i = 1, 2)$, our comparative statics yields

$$\frac{dx_0}{d\tau} = \frac{dy_0}{d\tau} = 3\frac{dx_i}{d\tau} = 3\frac{dy_i}{d\tau} = \frac{512}{729 - 2252\tau} > 0, \tag{7}$$

where the inequality is due to (6) . This result indicates that the public firm locates slightly away from the cen-

Figure 2. (Full agglomeration): The overlapping stars mean the agglomeration of the private firms. The arrows depict the movements for the firms when τ increases.

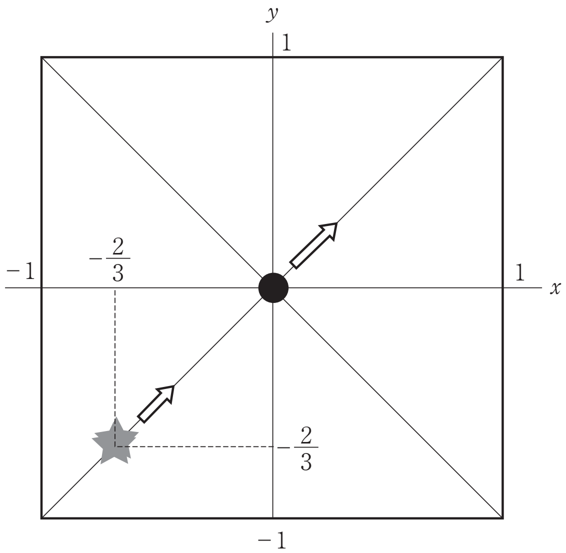


Table 1. The location under full agglomeration.

τ	0.01	0.02	0.03	0.04	0.0417=1/24
$x_0=y_0$	0.0036	0.0075	0.0116	0.0161	0.0168
$x_1=y_1=x_2=y_2$	−0.6655	−0.6642	−0.6628	−0.6613	−0.6611

ter to a point on a diagonal of L, while the private firms slightly approach the public firm on the same diagonal (Figure 2).

Unfortunately, the complexity of the equations prevents us from obtaining an analytical solution. Thus, we conduct a numerical analysis, and the computation yields the following result (Table 1). Under the restriction by (6), the equilibrium location is quite near the location in the case of $a\to\infty$. And each decrease in τ by 0.01 leads an increase in x_0 and y_0 by approximately 0.004 and an increase in x_1 (y_1, x_2, y_2), by approximately 0.0013, the ratio of which is consistent with the comparative statics in (7).

The intuition behind the result is as follows. The greater τ becomes, the more the public firm cares about the profits of the private firms. The movement of the public firm along the diagonal is the best way to separate from the private firms, with the loss of the consumer surplus being minimized because the consumer surplus is dependent only on the distance between the public firm and the center. Then, once the public firm goes away from the center, the private firms have incentives to approach the center due to relaxed competition.

4.3 Partial agglomeration

We can guess from Lemma 1 that if the public firm locates on either the x-axis or the y-axis, then there is an equilibrium where the private firms are symmetric with regard to that axis. Unfortunately, because the analytical insolvability continues, we will repeat a similar analysis as in full agglomeration. Without loss of generality, we only deal with the case where the public firm locates on the y-axis ($x_0=0$).

First of all, the limit case of $a\to\infty$ yields the following outcome.

Proposition 3 *Under the scheme of partial agglomeration, we have*

$$\lim_{a\to\infty}(x_0, y_0, x_1, y_1, x_2, y_2)=(0, 0, -2/3, -2/3, 2/3, -2/3).$$

Figure 3 represents this case. Furthermore, we proceed to an analysis in the vicinity of the limit case above. Holding $x_0=0$, a comparative statics that is evaluated at $(y_0, x_1, y_1, x_2, y_2)=(0, -2/3, -2/3, 2/3, -2/3)$ yields

$$\frac{dy_0}{d\tau}=\frac{512}{729-1996\tau}>0,$$
$$\frac{dx_1}{d\tau}=-\frac{dx_2}{d\tau}=\frac{1024}{2187-5998\tau}>0,$$

Figure 3. (Partial agglomeration): The locations of the public firm (the circle) and the private firms (the two stars). The arrows represent the movements of the firms when τ increases.

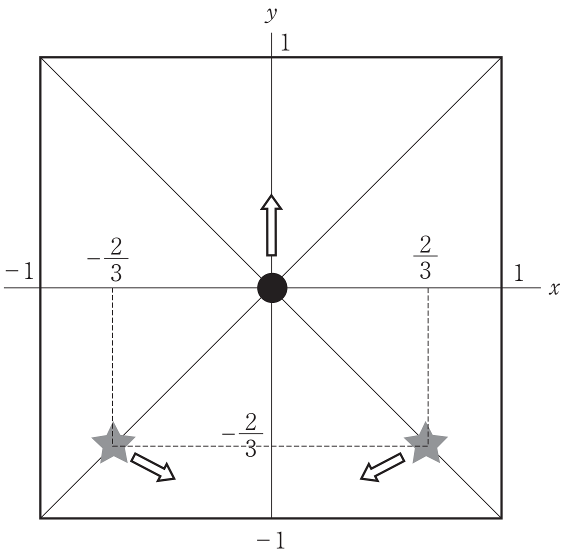


Table 2. The location under partial agglomeration.

τ	0.01	0.02	0.03	0.04	0.0417=1/24
y_0	0.0036	0.0074	0.0115	0.0158	0.0165
$x_1=-x_2$	−0.6642	−0.6617	−0.6589	−0.6559	−0.6553
$y_1=y_2$	−0.6679	−0.6691	−0.6704	−0.6717	−0.6719

$$\frac{dy_1}{d\tau}=\frac{dy_2}{d\tau}=\frac{512}{2187-5998\tau}>0,$$

where the inequalities are due to (6). Note that

$$\frac{dy_0}{d\tau}:\frac{dx_1}{d\tau}:\frac{dy_1}{d\tau}=3:2:-1. \quad (8)$$

When the public firm moves slightly northward from the center, firm 1 moves southeastward from $(x_1, y_1)=(-2/3, -2/3)$ (Figure 3). A numerical analysis confirms a successive movement as follows (Table 2). Furthermore, we can confirm that the magnitude of movement is consistent with the comparative statics in (8) as in full agglomeration.

The movement is quite different from that in full agglomeration, where the public firm just approaches the private firms to push them toward the edges. In partial agglomeration, the private firms do move away from the public firm in one dimension, although they approach it in the other dimension. The market boundary is a key to understanding what occurs.

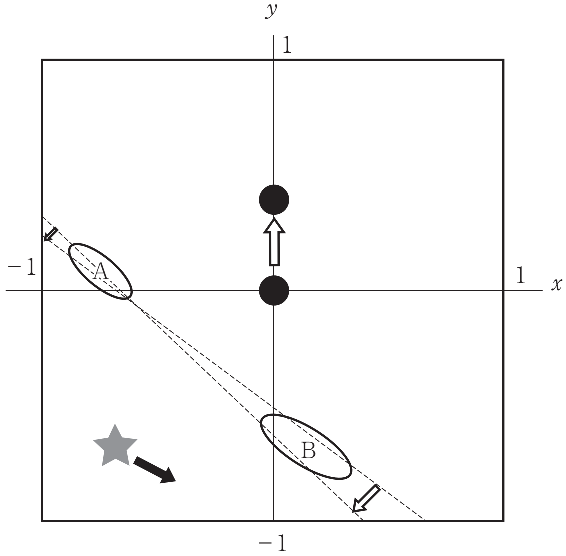
Suppose that $(x_0, y_0, x_1, y_1)=(0, 0, -2/3, -2/3)$ as an initial state (Figure 4). If the public firm moves slightly northward from the center $(x_0, y_0)=(0, \varepsilon)$, the boundary rotates counterclockwise a little bit. Hence, the northwestern markets become more competitive (near domain A in Figure 4). On the other hand, the southeastern markets become less competitive (near domain B in Figure 4). Therefore, firm 1 has an incentive to move southeastward, by which the firm can avoid keen competition.

Recall that Tabuchi (1994) and Irmen and Thisse (1998) show that differentiation is sufficient only in one dimension. Similarly, competition against the public sector is sufficient only in one dimension for private firms, and in the other dimension private firms should seek their niches.

5. Concluding remarks

We have analyzed an extended model of spatial mixed oligopoly. Some of our results are similar to those found in a linear city of MM (central symmetry and full agglomeration). As a new result, we have partial agglomeration, where private firms differentiate in only one dimension. Furthermore, the reactions of the firms to a change in transport costs are analyzed. When the public firm approaches the private firms, the private firms differentiate more in one dimension,

Figure 4. The locational adjustment of the private firm. The dotted lines show the market boundaries.



whereas they differentiate less in the other dimension.

We could extend the analysis by considering more than two private firms. Although the completion of analysis may not be an easy task, we could predict the answer. For example, let the number of firms, n, equal 4, and suppose that a public firm chooses the center. Then, it is likely that four private firms are dispersed at (k, k), $(k, -k)$, $(-k, k)$, $(-k, -k)$, where k is a constant $(0<k<1)$. Another possibility is that two firms locate at (k, k) and the other firms choose $(-k, -k)$. However, if n is odd, such symmetry would break down. Hence, this extension is quite complicated. Furthermore, more extension in dimensionality also causes complexity. Let these be the tasks of future research.

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Appendix

A. Proof of Lemma 1

Proof. Differentiating the social surplus with regard to x_0 and evaluating it at $x_0=0$, we have

$$\left.\frac{\partial ss}{\partial x_0}\right|_{x_0=0} = -\frac{32t^2}{3b}(x_1+x_2).$$

The first-order condition requires $x_1+x_2=0$.

Conversely, assume that $x_1+x_2=0$. Then, we have

$$\left.\frac{\partial ss}{\partial x_0}\right|_{x_2=-x_1} = -\frac{8t^2}{b}(a/t+4x_1^2+2y_1^2+2y_2^2-5x_0^2-5y_0^2-4)x_0.$$

Because $a/t>24$ from (2) , the value of the parenthesis is positive. Therefore, ss is maximized at $x_0=0$.
Hence, the public firm locates at $x_0=0$ if and only if $x_1+x_2=0$. Due to symmetry, we can apply identical analysis with regard to y-coodinate.

B. Proof of Proposition 1

Proof. Under the scheme of central symmetry, we immediately have that ss is maximized at $(x_0, y_0)=(0, 0)$ from Lemma 1. Hence, we can set $(x_0, y_0)=(0, 0)$. Due to symmetry, we focus on firm 1. Without loss of generality, we assume that $-1\leq y_1\leq x_1\leq 0$. Let $\frown{L}$, $\smile{L}\subset L$ be the domains such that

$$\frown{L}=\{(x, y) : -1\leq y\leq x\leq 0,\ (x-1)^2+(y+1)^2\geq 2\},$$
$$\smile{L}=\{(x, y) : -1\leq y\leq x\leq 0,\ (x-1)^2+(y+1)^2\leq 2\},$$

Then, we can find that the market boundary of $d_0(x, y)=d_1(x, y)$ intersects the lines of $x=-1$ and $y=-1$ if firm 1 locates in $\frown{L}$, whereas the boundary intersects the lines of $x=1$ and $x=-1$ if firm 1 locates in $\smile{L}$. We will investigate the properties of profit function in each case, and will obtain an optimal location by synthesis at the end.
First, we seek a maximizer in $\frown{L}$. The profit function of firm 1 is given by

$$\Pi_1=\frac{t^2[(x_0+1)^2+(y_0+1)^2-(x_1+1)^2-(y_1+1)^2]^4}{48b(x_0-x_1)(y_0-y_1)}$$

with $(x_0, y_0)=(0, 0)$. The first-order conditions yield the following, simplified equations:

$$6x_1+7x_1^2-2y_1-y_1^2=0,\ 6y_1+7y_1^2-2x_1-x_1^2=0.$$

The solution is only $(x_1, y_1)=(-2/3, -2/3)$, which also satisfies the second-order condition. Furthermore, we can readily show that no corner solutions exist. Hence, $(x_1, y_1)=(-2/3, -2/3)$ is the unique maximizer in $\widehat{L}$.

Second, we seek a maximizer in $\breve{L}$. By tedious calculations, we get the profit function of firm 1, and we can show that there is no interior solution that satisfies the first-order conditions in $\breve{L}$. Also, we can readily show that no corner solutions exist at each point on each edge. Thus, there is a maximizer on $(x-1)^2+(y+1)^2=2$ in $\breve{L}$. However, this is never the global maximizer because we can easily confirm the differentiability of the profit function at each point on $(x-1)^2+(y+1)^2=2$ despite the change in the definition of the function between in $\widehat{L}$ and in $\breve{L}$.

The synthesis shows that $(x_1, y_1)=(-2/3, -2/3)$ is the unique optimal location when $(x_0, y_0)=(0, 0)$. By symmetry, the profit of firm 2 is maximized at $(x_2, y_2)=(2/3, 2/3)$ under $(x_0, y_0)=(0, 0)$. Therefore, the location pair of $(x_0, y_0, x_1, y_1, x_2, y_2)=(0, 0, -2/3, -2/3, 2/3, 2/3)$ is the unique equilibrium under central symmetry.

Notes:

1) See also, among others, DeFraja and Delbono (1989) for an excellent, comprehensive analysis in the literature.
2) There are several papers with a different focus of interest in mixed oligopoly with location choice. See Matsushima and Matsumura (2006) and Heywood and Ye (2009) with foreign firms, and Li (2006) with multi-plant firms. Lu (2006) deals with mill pricing, and Matsumura and Matsushima (2003) study sequential choice of location.
3) Without a public firm, Pal (1998) and Matsushima (2001) show that such a full agglomeration of private firms does not occur in the circular space.
4) Ago (2008) also showed the central agglomeration is a unique equilibrium when space is a rectangle.
5) Recall that we omit other symmetric equilibria like $(x_0, y_0, x_1, y_1, x_2, y_2)=(0, 0, 2/3, -2/3, -2/3, 2/3)$ because it is essentially the same equilibrium in this proposition.

References

[1] Ago, T., 2008. Cournot competition in a two-dimensional city. TCUE Discussion Paper Series, No. 08-01. (http://www1.tcue.ac.jp/home1/c-gakkai/dp/dp08-01.pdf)

[2] Anderson, S. P., Neven, D. J., 1991. Cournot competition yields spatial agglomeration. International Economic Review 32, 793–808.

[3] Berenguer-Maldonado, M. I., Carbó-Valverde S., Fortes-Escalona, M. Á., 2005. Cournot competition in a two dimensional circular city. The Manchester School 73, 40–49.

[4] Cremer, H., Marchand, M., Thisse, J. -F., 1991. Mixed oligopoly with differentiated products. International Journal of Industrial Organization 9, 43–53.

[5] d'Aspremont, C., Gabszewicz, J. J., Thisse, J. -F., 1979. On hotelling's 'stability in competition.' Econometrica 47, 1045–1050.

[6] DeFraja, G., Delbono, F., 1989. Alternative strategies of a public enterprise in oligopoly. Oxford Economic Papers 41, 302–311.

[7] Hamilton, J. H., Thisse, J. -F., Weskamp, A., 1989. Spatial discrimination, Bertrand vs. Cournot in a model of location choice. Regional Science and Urban Economics 19, 87–102.

[8] Heywood, J. S., Ye, G., 2009. Mixed oligopoly and spatial price discrimination with foreign firms. Regional Science and Urban Economics 39, 592–601.

[9] Hotelling, H., 1929. Stability in competition. Economic Journal 39, 41–57.

[10] Irmen, A., Thisse, J. -F., 1998. Competition in multi-characteristics spaces: Hotelling was almost right. Journal of Economic Theory 78, 76–102.

[11] Li, C., 2006. Location choice in a mixed oligopoly. Economic Modelling 23, 131–141.

[12] Lu, Y., 2006. Hotelling's location model in mixed duopoly. Economics Bulletin 8, 1–10.

[13] Matsumura, T., Matsushima, N., 2003. Mixed duopoly with product differentiation: sequential choice of location. Australian Economic Papers 42, 18–34.

[14] Matsushima, N., 2001. Cournot competition and spatial agglomeration revisited. Economics Letters 73, 175–177.

[15] Matsushima, N., Matsumura, T., 2003. Mixed oligopoly and spatial agglomeration. Canadian Journal of Economics 36, 62–87.

[16] Matsushima, N., Matsumura, T., 2006. Mixed oligopoly, foreign firms, and location choice. Regional Science and Urban Economics 36, 753–772.

[17] Merrill, W. C., Schneider, N., 1966. Government firms in oligopoly industries: a short-run analysis. The Quarterly Journal of Economics 80, 400–412.

[18] Pal, D., 1998. Does Cournot competition yield spatial agglomeration? Economics Letters 60, 49–53.

[19] Tabuchi, T., 1994. Two-stage two-dimensional spatial competition between two firms. Regional Science and Urban Economics 24, 207–227.